**One dimensional longitudinal advection-diffusion equation in rivers**

Conservation of solute in 1D (Jamali et al. 2005) is given by:

**Crank Nicolson finite difference discretization**

**Formulas for longitudinal diffusion in rivers**

|  |  |
| --- | --- |
| Reference | Formula |
| Fischer et al. (1979) |  |
| Seo and Cheong (1998) |  |
| Kashefipour and Falconer (2002) |  |
| Disley et al. (2015) |  |
| Zeng and Huai (2014) |  |
| Tayfur and Singh (2005) | ANN model, dimensional data |
| Kargar et al. (2020) | Model tree: |

**Database**

Database is assembled from Carr & Rehmann (2005), Kashefipour & Falconer (2002), Deng et al. (2002), Zeng & Huai (2014), Seo & Cheong (1998), Tayful & Singh (2005) and by indirect referencing to Nordin & Sabol (1974), Godfrey and Frederick (1970), Yotsukura et al. (1970), & McQuivey & Keefer (1974). The database is composed of 180 data. Many of the data points are repeated across the references as they are repeatedly cited by various authors, therefore many data duplicates were found & manually removed.

Table 1. Database of longitudinal dispersion coefficient in rivers as a function of river width, flow depth, flow velocity and shear velocity

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| B (m) | H (m) | u (m/s) | u\* (m/s) | K (m2/s) |  | B (m) | H (m) | U (m/s) | u\* (m/s) | K (m2/s) |
| 1.4 | 0.19 | 0.38 | 0.11 | 9.6 |  | 20 | 0.52 | 0.43 | 0.069 | 16.3 |
| 1.5 | 0.14 | 0.33 | 0.1 | 1.9 |  | 20 | 0.4 | 0.19 | 0.18 | 6.5 |
| 3.6 | 0.2 | 0.15 | 0.1 | 1.9 |  | 20 | 0.42 | 0.29 | 0.045 | 13.9 |
| 3.6 | 0.27 | 0.23 | 0.12 | 2.4 |  | 20 | 1.2 | 0.45 | 0.099 | 32.5 |
| 5 | 0.28 | 0.26 | 0.21 | 7.2 |  | 20 | 1.92 | 0.62 | 0.123 | 11.5 |
| 5.1 | 0.25 | 0.14 | 0.09 | 2.5 |  | 20 | 0.45 | 0.26 | 0.08 | 6 |
| 5.5 | 0.3 | 0.15 | 0.09 | 3 |  | 21 | 0.48 | 0.29 | 0.09 | 7.4 |
| 5.6 | 0.31 | 0.26 | 0.09 | 2.4 |  | 21 | 0.31 | 0.98 | 0.11 | 8.2 |
| 7.6 | 3.45 | 0.68 | 0.047 | 0.5 |  | 21 | 0.48 | 0.52 | 0.069 | 25.9 |
| 9 | 0.3 | 0.37 | 0.15 | 8.4 |  | 21 | 0.5 | 0.13 | 0.037 | 12.8 |
| 9.9 | 0.83 | 0.46 | 0.09 | 5.5 |  | 21 | 0.5 | 0.54 | 0.03 | 501 |
| 9.9 | 0.92 | 0.52 | 0.1 | 5.1 |  | 21 | 0.93 | 0.36 | 0.035 | 24.2 |
| 10 | 0.35 | 0.53 | 0.17 | 12.4 |  | 22 | 0.56 | 0.38 | 0.09 | 10.2 |
| 10 | 0.63 | 0.55 | 0.3 | 13.5 |  | 22 | 0.61 | 0.08 | 0.042 | 12.8 |
| 10.8 | 0.2 | 0.41 | 0.13 | 2.4 |  | 22 | 1.58 | 0.31 | 0.058 | 6.5 |
| 11 | 0.52 | 0.21 | 0.075 | 17.5 |  | 24 | 1.6 | 0.67 | 0.04 | 6 |
| 11 | 0.21 | 0.49 | 0.14 | 6.1 |  | 24 | 1.56 | 0.71 | 0.043 | 9.6 |
| 11 | 0.29 | 0.35 | 0.058 | 2.7 |  | 24 | 0.98 | 0.59 | 0.098 | 102 |
| 11.2 | 0.24 | 0.66 | 0.16 | 5.4 |  | 24 | 0.71 | 0.52 | 0.081 | 25.6 |
| 11.4 | 0.75 | 0.41 | 0.061 | 8 |  | 25 | 1.6 | 0.66 | 0.04 | 5.9 |
| 11.6 | 0.4 | 0.22 | 0.087 | 1.9 |  | 25 | 0.45 | 0.41 | 0.081 | 25.9 |
| 11.9 | 0.66 | 0.43 | 0.085 | 20.9 |  | 25 | 0.56 | 1.01 | 0.137 | 13.9 |
| 12.2 | 0.51 | 0.23 | 0.03 | 14.7 |  | 25 | 1.21 | 0.73 | 0.084 | 27 |
| 12.5 | 0.26 | 0.31 | 0.04 | 7 |  | 25 | 1.38 | 0.77 | 0.091 | 20.5 |
| 12.8 | 0.3 | 0.42 | 0.057 | 17.5 |  | 25 | 1.4 | 0.78 | 0.091 | 15.5 |
| 13 | 0.26 | 0.31 | 0.044 | 7 |  | 25 | 1.57 | 0.83 | 0.096 | 18 |
| 13 | 0.6 | 0.48 | 0.24 | 6.8 |  | 26 | 0.9 | 0.34 | 0.07 | 32.5 |
| 13 | 0.81 | 0.37 | 0.081 | 13.9 |  | 26 | 0.94 | 0.34 | 0.067 | 27.6 |
| 13.4 | 0.81 | 0.37 | 0.08 | 13.9 |  | 27 | 1.1 | 0.44 | 0.007 | 24.6 |
| 13.7 | 0.85 | 1.29 | 0.55 | 2.9 |  | 29 | 0.61 | 0.35 | 0.07 | 10.7 |
| 14.2 | 0.5 | 0.13 | 0.037 | 12.8 |  | 30 | 1.08 | 0.36 | 0.048 | 0.5 |
| 14.5 | 0.31 | 0.25 | 0.062 | 1.9 |  | 30 | 1.1 | 0.38 | 0.025 | 35.9 |
| 14.9 | 0.59 | 0.27 | 0.08 | 10.3 |  | 31 | 1.43 | 0.13 | 0.041 | 24.2 |
| 15.7 | 0.23 | 0.36 | 0.039 | 69 |  | 32 | 0.76 | 0.36 | 0.053 | 44 |
| 15.8 | 0.39 | 0.32 | 0.06 | 9.3 |  | 32 | 0.5 | 0.24 | 0.038 | 52.2 |
| 15.8 | 0.41 | 0.37 | 0.06 | 13.9 |  | 33 | 0.3 | 0.43 | 0.05 | 9.3 |
| 15.9 | 0.22 | 0.39 | 0.05 | 7.1 |  | 33 | 1.4 | 0.2 | 0.031 | 54.7 |
| 15.9 | 0.22 | 0.39 | 0.053 | 7.1 |  | 34 | 0.78 | 0.19 | 0.049 | 10.7 |
| 15.9 | 0.49 | 0.21 | 0.08 | 19.5 |  | 34 | 0.85 | 0.16 | 0.06 | 9.5 |
| 16 | 0.43 | 0.37 | 0.056 | 13.9 |  | 34 | 0.85 | 0.15 | 0.055 | 9.5 |
| 16 | 0.49 | 0.26 | 0.08 | 9.5 |  | 34 | 2.5 | 0.13 | 0.008 | 1.7 |
| 16.7 | 0.49 | 0.2 | 0.08 | 16.8 |  | 34 | 2.47 | 0.82 | 0.18 | 65 |
| 16.8 | 0.5 | 0.24 | 0.08 | 24.6 |  | 35 | 2.5 | 0.037 | 0.002 | 0.2 |
| 17.2 | 0.22 | 0.52 | 0.09 | 7.13 |  | 35 | 2.5 | 0.107 | 0.006 | 1.4 |
| 17.4 | 1.23 | 0.04 | 0.050 | 14.7 |  | 35 | 0.32 | 0.21 | 0.04 | 4.7 |
| 17.5 | 0.45 | 0.32 | 0.024 | 5.8 |  | 35 | 0.98 | 0.21 | 0.04 | 39.5 |
| 18.3 | 0.38 | 0.15 | 0.116 | 20.7 |  | 36 | 0.58 | 0.3 | 0.05 | 8.1 |
| 18.3 | 0.84 | 0.52 | 0.1 | 21.4 |  | 37 | 0.91 | 0.42 | 0.67 | 39.5 |
| 18.6 | 0.39 | 0.14 | 0.12 | 9.9 |  | 37 | 0.45 | 0.32 | 0.05 | 13.9 |
| 19 | 0.4 | 0.16 | 0.116 | 9.9 |  | 37 | 0.9 | 0.13 | 0.05 | 15.5 |
| 19.4 | 0.26 | 0.74 | 0.09 | 5.5 |  | 37 | 0.81 | 0.29 | 0.07 | 23.2 |
| 19.5 | 1.2 | 0.45 | 0.093 | 32.5 |  | 41 | 0.41 | 0.23 | 0.04 | 66.5 |
| 19.6 | 0.8 | 0.49 | 0.1 | 20.8 |  | 42 | 1.04 | 0.07 | 0.090 | 10.3 |
| 19.8 | 0.41 | 0.29 | 0.04 | 13.9 |  | 42 | 0.69 | 0.23 | 0.064 | 40.8 |

Table 1. Continued

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| B (m) | H (m) | u (m/s) | u\* (m/s) | K (m2/s) |  | B (m) | H (m) | U (m/s) | u\* (m/s) | K (m2/s) |
| 42 | 0.8 | 0.42 | 0.068 | 30.2 |  | 86 | 2.81 | 1.24 | 0.51 | 1798 |
| 43 | 1.13 | 0.63 | 0.081 | 53.3 |  | 86 | 2.93 | 1.2 | 0.53 | 153 |
| 44 | 1.37 | 0.99 | 0.142 | 184 |  | 92 | 2.44 | 0.52 | 0.094 | 167 |
| 47 | 0.86 | 0.28 | 0.07 | 13.9 |  | 93 | 0.71 | 0.16 | 0.05 | 41.4 |
| 48 | 0.87 | 0.44 | 0.07 | 37.2 |  | 98 | 1.15 | 0.32 | 0.058 | 120 |
| 49 | 1.2 | 0.21 | 0.07 | 14.8 |  | 100 | 4.4 | 0.029 | 0.002 | 0.2 |
| 49 | 0.55 | 0.26 | 0.052 | 37.8 |  | 102 | 4.4 | 0.17 | 0.008 | 22.4 |
| 49 | 8.07 | 0.27 | 0.019 | 3 |  | 104 | 2.04 | 0.56 | 0.05 | 316 |
| 50 | 0.41 | 0.15 | 0.081 | 29.3 |  | 106 | 6.1 | 0.79 | 0.089 | 181 |
| 50 | 0.95 | 0.32 | 0.075 | 29.6 |  | 116 | 1.65 | 0.58 | 0.054 | 131 |
| 51 | 0.42 | 0.46 | 0.046 | 20.9 |  | 120 | 2 | 0.64 | 0.05 | 67 |
| 51 | 0.65 | 0.62 | 0.04 | 29.6 |  | 127 | 4.75 | 0.64 | 0.08 | 669 |
| 53 | 2.41 | 0.66 | 0.107 | 36.9 |  | 152 | 3.66 | 0.45 | 0.057 | 228 |
| 53 | 2.09 | 0.79 | 0.11 | 46.5 |  | 155 | 1.74 | 0.47 | 0.036 | 178 |
| 58 | 2.45 | 0.75 | 0.104 | 40.5 |  | 158 | 4.3 | 0.19 | 0.007 | 48.9 |
| 59 | 0.72 | 0.37 | 0.07 | 32 |  | 160 | 2.32 | 1.06 | 0.054 | 309 |
| 59 | 1.1 | 0.88 | 0.12 | 41.8 |  | 162 | 0.4 | 0.34 | 0.02 | 44 |
| 59 | 2.13 | 0.86 | 0.1 | 53.9 |  | 162 | 3.96 | 0.29 | 0.06 | 131 |
| 60 | 0.95 | 0.46 | 0.092 | 47 |  | 167 | 0.2 | 0.47 | 0.159 | 43.2 |
| 63 | 1 | 0.32 | 0.094 | 22 |  | 176 | 3.4 | 1.61 | 0.082 | 966 |
| 64 | 0.46 | 0.1 | 0.056 | 29.3 |  | 183 | 3.29 | 1.55 | 0.079 | 1486 |
| 64 | 0.76 | 0.67 | 0.268 | 34.8 |  | 183 | 2.23 | 0.93 | 0.07 | 465 |
| 66 | 1.13 | 0.39 | 0.075 | 32.5 |  | 183 | 5.7 | 0.11 | 0.02 | 13.3 |
| 67 | 0.55 | 0.35 | 0.044 | 30.2 |  | 194 | 6.3 | 0.22 | 0.039 | 538 |
| 67 | 0.98 | 0.88 | 0.11 | 41.8 |  | 197 | 3.11 | 1.53 | 0.078 | 892 |
| 69 | 2.16 | 1.55 | 0.168 | 163 |  | 201 | 3.56 | 1.28 | 0.084 | 837 |
| 70 | 2.35 | 0.43 | 0.1 | 111 |  | 202 | 4.6 | 0.18 | 0.036 | 49.1 |
| 72 | 8.2 | 1.2 | 0.337 | 243 |  | 203 | 1.35 | 0.39 | 0.065 | 92.9 |
| 75 | 1.6 | 0.22 | 0.99 | 17 |  | 229 | 3.4 | 1.24 | 0.082 | 310 |
| 76 | 1.95 | 0.74 | 0.138 | 88.9 |  | 230 | 3.5 | 1.08 | 0.085 | 455 |
| 76 | 1.2 | 1.41 | 0.058 | 116 |  | 254 | 0.81 | 0.48 | 0.072 | 45.1 |
| 78 | 1.2 | 1.42 | 0.026 | 326 |  | 259 | 3.3 | 0.17 | 0.017 | 24.2 |
| 80 | 2.74 | 0.034 | 0.0024 | 22.3 |  | 300 | 0.3 | 1 | 0.029 | 350 |
| 80 | 2.74 | 0.14 | 0.0097 | 34.9 |  | 300 | 0.4 | 0.97 | 0.032 | 228 |
| 85 | 2.6 | 0.69 | 0.06 | 52 |  | 537 | 8.9 | 1.51 | 0.097 | 374 |
| 85 | 2.38 | 1.74 | 0.153 | 464 |  | 711 | 19.9 | 0.56 | 0.041 | 237 |

Figure 1. Dimensionless graph of vs.

Figure 2. Dimensional graph of K vs. river width B

Figure 3. Dimensional graph of K vs. flow depth u

**Error estimate via discrepancy ratio**

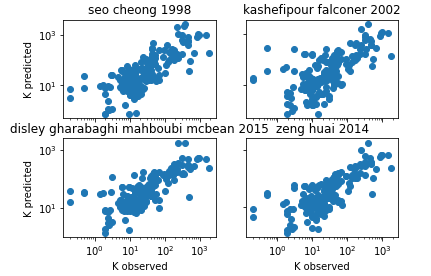


Figure 3. Observation vs. prediction of longitudinal dispersion (K) with 4 methods

**Notation**

|  |  |
| --- | --- |
| h (m) | flow depth |
| K (m2/s) | longitudinal dispersion coefficient |
| R (m) | hydraulic radius |
| S | Slope of energy gradient or bed slope |
| u (m/s) | cross-sectional average velocity |
| u\* (m/s) | shear velocity |
| w (m) | channel width |
| C (mg/L) | cross-sectional average concentration |
| A (m2) | flow cross-sectional area |
| x (m) | longitudinal coordinate |

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